ON THE THEORY OF STABILITY OF A FLUID CONDUCTING CYLINDER IN A MAGNETIC FIELD

(K TEORII USTOICHIVOSTI ZHIDKOGO PROVODIASHCHEGO TSILINDRA V MAGNITNOM POLE)

PMM Vol.26, No.5, 1962, pp. 877-884

Iu. V. VANDAKUROV
(Leningrad)

(Received July 7, 1962)

The problem of stability of a cylindrical plasma filament contained by a magnetic field has been considered in the approximation of a perfectly conducting plasma in [1-7] and a number of other papers. The study of configurations with surface currents, reported in [1-2], was followed in certain cases by studies of distributed currents (on an incompressible cylinder model) [2-5]. The criterion of stability relative to perturbations of local type was obtained by Suydam [6] on the basis of the energy principle. It was shown in [5] that the fulfilment of this criterion still did not guarantee stability with regard to all types of perturbation. Local perturbations were studied also by Rosenbluth [7] and others.

The problem of stability with allowance for the finite conductivity of the medium is more complicated. The case of a poorly conducting cylinder can be studied comparatively simply. For a cylinder with uniform conductivity the calculations were carried out in [8-10].

In the present paper the stability of a plasma filament in the form of a tube situated in a vacuum or an incompressible medium is studied on the basis of a hollow incompressible cylinder model. It is assumed that in the equilibrium state in the conducting layer there are both (azimuthal and axial) components of the magnetic field. The study of stability is carried out on the basis of the linearized equations of magnetohydrodynamics by the method of normal oscillations. The limiting cases are considered of infinitely large and small conductivity of the hollow cylinder. In the first case the dispersion equation is expressed in quadratures for long-wave perturbations leading to spiral twisting of the cylinder, under the conditions that the longitudinal magnetic field is almost uniform. In particular, for a solid conducting cylinder the criterion of stability turns out to be independent of the distribution of longitudinal current. In the case of small conductivity the characteristic oscillations are absent if the density distribution of volume current is continuous. They arise only in the presence of discontinuities in the distribution of conductivity.

In Section 1 is formulated the problem for the case of arbitrary degree of conductivity of the medium. Sections 2 and 3 are devoted to a study of stability in the approximation of a perfectly conducting fluid. Stability of a poorly conducting cylinder is considered in Section 4.

1. Formulation of the problem. Let us denote the internal and external radii (see figure) of the conducting layer *0* by r_1 and r_2. We shall assume that inside the tube there is a perfect conductor *3* of radius $\alpha_1 r_1$, through which flows a current, whilst the outside of the region under consideration is enclosed by a perfectly conducting cylinder *4* of radius $\alpha_2 r_2$ (the layers *1* and *2* are nonconducting).

Suppose that in the equilibrium state the velocity $\mathbf{v} = 0$, whilst the distribution of magnetic field has the form

$$\frac{\mathbf{H}}{H_0} = \begin{cases} \mathbf{i}_\varphi \dfrac{r_1^2 g_1}{r_0 r} + \mathbf{i}_z h_1 & (\alpha_1 r_1 \leqslant r \leqslant r_1) \\ \mathbf{i}_\varphi \dfrac{r}{r_0}\, g(r) + \mathbf{i}_z h(r) & (r_1 \leqslant r \leqslant r_2) \\ \mathbf{i}_\varphi \dfrac{r_2^2 g_2}{r_0 r} + \mathbf{i}_z h_2 & (r_2 \leqslant r \leqslant \alpha_2 r_2) \end{cases} \qquad (1.1)$$

$$g = \frac{r_0 H_\varphi}{r H_0}, \quad h = \frac{H_z}{H_0}, \quad g_j = g(r_j), \quad h_j = h(r_j), \quad H_0 = (H_\varphi)_{r=r_0}$$

Here $\mathbf{i}_r$, $\mathbf{i}_\varphi$, $\mathbf{i}_z$ are base vectors in a cylindrical system of coordinates; g and h are arbitrary functions of r (for a medium of finite conductivity in the region $r_1 < r < r_2$ the functions g, h, $g' = dg/dr$, $h' = dh/dr$ must be continuous); r_0 is a certain intermediate radius (in what follows we shall generally take $r_0 = r_2$). In the region $r_1 < r < r_2$ the conductivity and density of the medium are equal to σ and ρ (ρ is constant), whilst the pressure

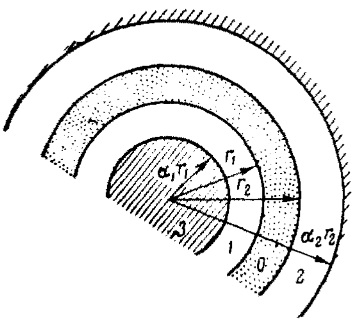
$$p(r) = p_1 + \frac{H_0^2}{8\pi}\left\{\frac{r_1^2 g_1^2 - r^2 g^2}{r_0^2} + h_1^2 - h^2 - \frac{2}{r_0^2}\int_{r_1}^{r} r g^2\, dr\right\} \qquad (1.2)$$

Let the density and pressure outside the conducting layer be $\chi_j \rho$ and p_j, where $j = 1$ if $r < r_1$, and $j = 2$ if $r > r_2$, whilst χ_j, p_j are constants.

The conductivity $\sigma(r)$ is connected with the field distribution $\mathbf{H}(r)$ by the relation

$$\frac{4\pi\sigma}{cH_0}\mathbf{E} = -\mathbf{i}_\varphi \frac{dh}{dr} + \mathbf{i}_z \frac{1}{r_0 r}\frac{dr^2 g}{dr} \quad \left(\mathbf{E} = \mathbf{i}_\varphi \frac{r_0 E_{\varphi_0}}{r} + \mathbf{i}_z E_{z0},\ E_{\varphi 0}, E_{z0} = \text{const}\right) \tag{1.3}$$

In the study of stability of a slowly moving cylinder, or in the case of a slowly changing magnetic field the electric field $\mathbf{E}$ in Equation (1.3) may be a more complicated function of r.



Suppose that a small perturbation is superimposed on the specified equilibrium distribution, so that the resultant quantities are: $\mathbf{H} + \mathbf{H}^*$, $p + p^*$, $\mathbf{v}^*$, where the asterisks denote the perturbations, which are functions of time and the coordinates φ, z in the form $\exp[i(\omega t + kz + m\varphi)]$.

We shall assume that $m \geqslant 0$, which can be ensured by suitable choice of the direction of the z-axis. Moreover the sign of the constant H_0 will depend upon the type of oscillation under examination. In view of the fact that only the sign of the product kh is important we can still choose $k \geqslant 0$, then instead of perturbations with negative k we shall need to consider the stability for a distribution with a changed sign in front of h.

Setting $\mathbf{v}^* = i\omega\boldsymbol{\xi}$ in the basic linearized system of equations of magnetohydrodynamics

$$-4\pi\rho\omega^2\boldsymbol{\xi} = -\nabla(4\pi p^* + \mathbf{H}\cdot\mathbf{H}^*) + (\mathbf{H}^*\cdot\nabla)\mathbf{H} + (\mathbf{H}\cdot\nabla)\mathbf{H}^* \tag{1.4}$$

$$\mathbf{H}^* = \operatorname{rot}(\boldsymbol{\xi}\times\mathbf{H}) - \frac{c^2}{4\pi i\omega}\operatorname{rot}\frac{1}{\sigma}\operatorname{rot}\mathbf{H}^*, \qquad \operatorname{div}\boldsymbol{\xi} = 0, \qquad \operatorname{div}\mathbf{H}^* = 0$$

we obtain

$$\frac{\Omega^2 H_0}{r_0}\boldsymbol{\xi} = -\nabla Q + is\mathbf{H}^* - 2\mathbf{i}_r g H_\varphi^* + [\mathbf{i}_\varphi(rg' + 2g) + \mathbf{i}_z r_0 h'] H_r^* \tag{1.5}$$

$$\mathbf{H}^* = \frac{isH_0}{r_0}\boldsymbol{\xi} - (\mathbf{i}_\varphi rg' + \mathbf{i}_z r_0 h')\frac{H_0\xi_r}{r_0} + \frac{r_0^2}{\Omega q^2}\left\{\nabla^2\mathbf{H}^* + \frac{\sigma'}{\sigma}\mathbf{i}_r\times\operatorname{rot}\mathbf{H}^*\right\} \tag{1.6}$$

$$\operatorname{div}\boldsymbol{\xi} = 0 \tag{1.7}$$

Here dashes denote differentiation with respect to r

$$Q = \frac{r_0}{H_0}(4\pi p^* + \mathbf{H}\cdot\mathbf{H}^*), \qquad \operatorname{div}\mathbf{H}^* = 0$$

$$\Omega = \frac{i\omega r_0 \sqrt{4\pi\rho}}{|H_0|}, \qquad q^2 = \frac{r_0\sigma |H_0|}{c^2}\sqrt{\frac{4\pi}{\rho}}, \qquad s = mg + kr_0 h$$

To determine the boundary conditions let us consider the region outside the conducting layer. Here the perturbation of the magnetic field is

$$\mathbf{H}^{*(j)} = \nabla\psi^{(j)} \qquad \left(\nabla^2\psi^{(j)} = 0, \ \left(\frac{\partial\psi^{(j)}}{\partial r}\right)_{r=\alpha_j r_j} = 0\right) \qquad (j = 1,2)$$

Hence it follows that

$$(\mathbf{i}_\varphi \nabla\psi^{(j)})_{r=r_j} = \frac{m}{kr_j}(\mathbf{i}_z\nabla\psi^{(j)})_{r=r_j} = \frac{i}{T_j}\left(\frac{\partial\psi^{(j)}}{\partial r}\right)_{r=r_j}$$

Here

$$T_j = \frac{r_j}{m}\left(\frac{d\ln\psi^{(j)}}{dr}\right)_{r=r_j} = \frac{kr_j\,[I_m'(kr_j)\,K_m'(k\alpha_j r_j) - K_m'(kr_j)\,I_m'(k\alpha_j r_j)]}{m\,[I_m(kr_j)\,K_m'(k\alpha_j r_j) - K_m(kr_j)\,I_{m'}(k\alpha_j r_j)]}$$

$$I_m'(z) = \frac{dI_m(z)}{dz} \qquad \left\{T_j \approx \frac{1-\alpha_j^2}{1+\alpha_j^2}\right\}$$

In the brace brackets is shown the expression for T_j when kr_j, $k\alpha_j r_j$ are small, and also when α_2 tends to infinity.

From the equation

$$\chi_j\rho\omega^2\xi^{(j)} = \nabla p^{*(j)}$$

we find that

$$(p^{*(j)})_{r=r_j} = \frac{\omega^2\rho_0\chi_j r_j}{mT_j}(\xi_r)_{r=r_j}$$

On the boundary $r = r_j$ the field and pressure must be continuous

$$\mathbf{H}(r_j) + \left(\xi_r\frac{d\mathbf{H}}{dr} + \mathbf{H}^*\right)_{r=r_j}, \qquad p(r_j) + \left(\xi_r\frac{dp}{dr} + p^*\right)_{r=r_j}$$

Hence it follows that

$$\left(\frac{\partial\psi^{(j)}}{\partial r}\right)_{r=r_j} = (H_r^*)_{r=r_j} = H_{rj}^*$$

The three other conditions give

$$mQ_j + \frac{r_j}{T_j}\left(\Omega^2\chi_j\frac{H_0\xi_{rj}}{r_0} - is_jH_{rj}^*\right) = 0 \tag{1.8}$$

$$H_{\varphi j}^* - \frac{i}{T_j} H_{rj}^* + (r_j g_j' + 2g_j) \frac{H_0 \xi_{rj}}{r_0} = 0 \tag{1.9}$$

$$H_{zj}^* + h_j' H_0 \xi_{rj} - \frac{ikr_j}{mT_j} H_{rj}^* = 0 \tag{1.10}$$

$$\left(g_j' = \left(\frac{dg}{dr}\right)_{r=r_j}, \ \xi_{rj} = (\xi_r)_{r=r_j} \ \text{etc.}\right)$$

Accordingly, the problem reduces to finding the root Ω of the determinant of the system (1.8) to (1.10), in which are substituted appropriate solutions of Equations (1.5) to (1.7). For unstable oscillations the real part of Ω must be positive ($\mathrm{Re}\,\Omega > 0$).

2. The approximation of perfect conductivity. When $q = \infty$, i.e. in the approximation of perfect conductivity of the medium, it is not difficult to obtain from (1.5) to (1.6) the equation

$$-\frac{H_0}{r_0} x\boldsymbol{\xi} = \mathbf{i}_r\left(Q' + \frac{m\mathfrak{Z}}{r} Q\right) + \mathbf{i}_\varphi i\left[\beta Q' + \frac{m(1-\delta^2)}{r} Q\right] + \mathbf{i}_z ik\,(1 - \beta^2 - \delta^2)Q \tag{2.1}$$

Here

$$\beta = \frac{2sg}{s^2 + \Omega^2}, \qquad \delta^2 = \frac{2rgg'}{s^2 + \Omega^2}, \qquad x = (1 - \beta^2 - \delta^2)(s^2 + \Omega^2)$$

Substitution of (2.1) in Equation (1.7) leads to the equation obtained in [4]

$$Q'' + \left(\frac{1}{r} - \frac{x'}{x}\right)Q' - \left\{k^2(1 - \beta^2 - \delta^2) + \frac{m\mathfrak{Z}x'}{rx} - \frac{m\mathfrak{Z}'}{r} + \frac{m^2(1-\delta^2)}{r^2}\right\}Q = 0 \tag{2.2}$$

By means of the substitutions

$$-\frac{r_0 Q}{H_0 \xi_r} = \frac{xQ}{Q' + \dfrac{m\mathfrak{Z}}{r} Q} = \frac{x\Psi}{\Psi'} = \Phi = \frac{r^2(s^2 + \Omega^2)}{m^2 + k^2r^2}\frac{\Lambda'}{\Lambda} \tag{2.3}$$

Equation (2.2) can be reduced to the following forms:

$$\Psi'' + \left(\frac{1 - 2m\mathfrak{Z}}{r} - \frac{x'}{x}\right)\Psi' - \frac{x(m^2 + k^2r^2)}{r^2(s^2 + \Omega^2)}\Psi = 0 \tag{2.4}$$

$$\Phi' + \frac{m^2 + k^2r^2}{r^2(s^2 + \Omega^2)}\Phi^2 - \frac{1 - 2m\mathfrak{Z}}{r}\Phi - x = 0 \tag{2.5}$$

$$\Lambda'' + \left[\frac{2ss'}{s^2 + \Omega^2} + \frac{2m\mathfrak{Z}}{r} + \frac{m^2 - k^2r^2}{r(m^2 + k^2r^2)}\right]\Lambda' - \frac{x(m^2 + k^2r^2)}{r^2(s^2 + \Omega^2)}\Lambda = 0 \tag{2.6}$$

The boundary conditions arise from the conditions of continuity of total pressure (hydrodynamic plus magnetic) on the surfaces $r = r_j$.

If there are no discontinuities in the field $\mathbf{H}(r)$ at the boundaries, then Formula (1.8) remains valid, or

$$mQ_j + \frac{r_j}{T_j}(\Omega^2\chi_j + s_j^2)\frac{H_0\xi_{rj}}{r_0} = 0 \tag{2.7}$$

To study the stability of a cylinder with surface currents let us consider such a distribution when $g(r)$ and $h(r)$ change rapidly in a thin layer at the surface, so that $g' \gg g/r$ and so on.

On the assumption that $s^2 + \Omega^2$ is not close to zero, Equation (2.5) is easily integrated. It is clear that $\Phi(r)$ cannot be such a rapidly varying function of r, that the second term in the left-hand side of (2.5) is important, and therefore in the region of the current layer

$$\Phi + rg^2 = \text{const} \tag{2.8}$$

Integrating again the r-component of (2.1) and using (2.8), we find that

$$\xi_r = \text{const} \tag{2.9}$$

The equations obtained are, of course, in accord with the results of the usual method of studying the stability of a filament with surface currents [1-2], in which the equations of magnetohydrodynamics are not solved in the region of the current layer, but the discontinuity in the tangential component of the steady field is allowed for in the boundary conditions. The case where zeros of the function $s^2 + \Omega^2$ are present requires special study (see [7]). For example, there remains the difficult question as to what extent Equations (2.8), (2.9) can be analytically continued in a region of arbitrary values of $s^2 + \Omega^2$.

Equations (2.8), (2.9) remain valid even for a compressible medium, when instead of (1.7) we use the equations of continuity and isentropic motion.

3. The approximation of perfect conductivity. Long-wave perturbations. Let us consider perturbations $m \neq 0$, the wave length of which is large in comparison with the radius r_2, so that $k^2 r_2^2 \ll 1$. From Formula (2.1) it follows that ξ_z is of order $kr_0\xi_r$, and therefore to an order of accuracy (k^2r^2) we shall have*

---

* Here we do not consider the case when there are points where $s^2 + \Omega^2 = 0$ in the interval under consideration. In the neighborhood of such a point there may be other important small terms, neglected in the basic equations of magnetohydrodynamics.

$$\xi_\varphi = \frac{i}{m}\frac{\partial r\xi_r}{\partial r}$$

The system (1.5), (1.6) for the variable $R = rH_0\xi_r/r_0$ enables one to obtain (when $q = \infty$)

$$r^2(s^2+\Omega^2)R'' + r(s^2+\Omega^2+2rss')R' -$$
$$- m^2\left[s^2+\Omega^2-2rgg'+\frac{2r}{m}(sg'+gs')\right]R = 0 \qquad (3.1)$$

Moreover

$$Q = -\frac{r(s^2+\Omega^2)}{m^2}R' + \frac{2sg}{m}R, \quad H_r^* = \frac{is}{r}R, \quad H_\varphi^* = -\frac{s}{m}R' - g'R \qquad (3.2)$$

Two examples, for which the solution of (3.1) can be expressed in terms of known functions, were studied in [5]. Here we consider the stability of configurations with approximately uniform longitudinal magnetic field, moreover we restrict ourselves to consideration of the most troublesome perturbations, leading to spiral twisting of the cylinder.

In particular, we shall assume that $m = 1$, whilst H_z' is of smaller order than kH_0. We shall assume also that the distribution of the field $\mathbf{H}(r)$ in the steady state is continuous. Taking account of the fact that $s' \approx g'$, from (3.1) we obtain

$$R = r\left\{C_1 + C_2\int_{r_1}^{r}\frac{dr}{r^3(s^2+\Omega^2)}\right\}, \qquad C_n = \text{const} \qquad (3.3)$$

Substitution of (3.3) in (2.7) leads to the dispersion equation

$$r_1r_2S_1S_2\int_{r_1}^{r_2}\frac{dr}{r^3(s^2+\Omega^2)} + \frac{r_1}{r_2}S_1 - \frac{r_2}{r_1}S_2 = 0 \qquad (3.4)$$

$$S_j = s_j^2 + \Omega^2 - 2s_jg_j - (s_j^2+\Omega^2\chi_j)\frac{1+\alpha_j^2}{1-\alpha_j^2}$$

For a solid cylinder $r_1 = 0$ and $S_1 = 0$. This means that, setting $r_0 = r_2$, we shall have $g_2 = 1$, $s_2 = 1 + kr_0h$

$$\Omega^2\left(1+\chi_2\frac{\alpha_2^2+1}{\alpha_2^2-1}\right) = 2s_2\left(1-\frac{\alpha_2^2s_2}{\alpha_2^2-1}\right) \qquad (3.5)$$

The frequency of the oscillations does not depend upon the distribution of the field H_φ and H_z with respect to the section of the cylinder. The only important quantity is the longitudinal magnetic field (if the latter is sufficiently great, so that kr_0h is of order unity). The the values of $\Omega^2 > 0$ there corresponds the range of instability

$$0 < 1 + kr_0 h < 1 - \frac{1}{\alpha_2^2}$$

For the case $g(r) \equiv 1$, $h = \text{const}$, $\chi_2 = 0$, this result can be obtained from the Formulas of [3].

For a solid cylinder of uniform density, the internal part of which $0 < r < r_1$ is nonconducting, in the absence of an internal conductor we have

$$\chi_1 = 1, \qquad \alpha_1 = 0, \qquad g_1 = 0$$

Since $S_1 = 0$, the dispersion equation (3.4) again reduces to Formula (3.5).

Let us consider again the stability of a tubular cylinder surrounded by a vacuum in the case when the longitudinal current is uniform across the section, when (with $r_0 = r_2$) we have

$$\chi_1 = \chi_2 = 0, \quad g = 1, \quad s = 1 + kr_0 h \approx \text{const}, \quad \alpha_1 = 0, \quad \alpha_2 = \infty$$

The conditions of equilibrium are satisfied if $8\pi p \ll H_z^2$, whilst $H_z(r)$ is a slowly decreasing function of (r). From Equation (3.4) we obtain

$$\Omega^2 = -\frac{2s}{r_2^2 - r_1^2} \{sr_2^2 \pm \sqrt{(r_2^2 - r_1^2)^2 + r_1^2 r_2^2 s^2}\} \qquad (3.6)$$

One of the solutions is unstable in the range

$$-\frac{1}{r_2}\sqrt{r_2^2 - r_1^2} < 1 + kr_0 h < 0$$

whilst the other is unstable in the range

$$0 < 1 + kr_0 h < \frac{1}{r_2}\sqrt{r_2^2 - r_1^2}$$

In accordance with the increased number of boundaries we obtain two ranges of instability.

4. The approximation of poor conductivity. In the case of poor conductivity of the medium, when $q \ll 1$, instead of Equation (1.6) we have

$$\nabla^2 \mathbf{H}^* + \frac{\sigma'}{\sigma} \mathbf{i}_r \times \operatorname{rot} \mathbf{H}^* = 0 \qquad (4.1)$$

For the components of $\mathbf{H}^*$ we obtain the system

$$\left(\nabla^2 - \frac{1}{r^2}\right) imH_r^* + \frac{2m^2}{r^2} H_\varphi^* = 0$$

$$\left(\frac{2}{r^2}+\frac{\sigma'}{r\sigma}\right) imH_r^* = -\left(\nabla^2-\frac{1}{r^2}\right) H_\varphi^* + \frac{\sigma'}{r\sigma}\frac{\partial r H_\varphi^*}{\partial r} \tag{4.2}$$

$$krH_z^* = i\,\frac{\partial r H_r^*}{\partial r} - mH_\varphi^* \tag{4.3}$$

Let us consider the stability with respect to long-wave perturbations ($k^2r^2 \ll 1$). Suppose again that $m \neq 0$ and h' is of order k.

From the system (4.2) we find that

$$rH_r^* = -\,i\,(D_1r^m - D_2r^{-m}), \qquad rH_\varphi^* = D_1r^m + D_2r^{-m}$$
$$D_n = \text{const} \tag{4.4}$$

We then obtain

$$\nabla^2 Q = \frac{2g'}{r}\,[(m-1)\,D_1r^m - (m+1)\,D_2r^{-m}]$$

$$Q = 2\,(m-1)\,D_1r^{-m}\int\limits_{r_1}^{r} gr^{2m-1}dr - 2\,(m+1)\,D_2r^m\int\limits_{r_1}^{r} gr^{-2m-1}dr + D_3r^m + D_4r^{-m} \tag{4.5}$$

$$\frac{\Omega^2H_0r\xi_r}{r_0} = 2m\,(m-1)\,D_1r^{-m}\int\limits_{r_1}^{r} gr^{2m-1}dr + D_1\,(s-2mg)\,r^m + 2m\,(m+1)\,D_2r^m\int\limits_{r_1}^{r} gr^{-2m-1}dr - D_2\,(s-2mg)\,r^{-m} - mD_3r^m + mD_4r^{-m} \tag{4.6}$$

Substituting the solutions in the conditions (1.8), (1.9) leads to a system, the vanishing of whose determinant gives the dispersion equation. The conditions (1.10) serve here for determining the arbitrary constants D_5 and D_6 arising in the following approximation with regard to the small parameter k^2r^2 in the expression for H_z^*. (If H_z^* were of order H_r^*/kr_0, then the dispersion equation would follow from the boundary conditions (1.10). It has, however, no solution.)

Let us write out the dispersion equation for the case of a solid filament of uniform density, in the internal portion of which ($r < r_1$) the field $H_\varphi = 0$ and $\sigma = 0$. Then $\chi_1 = 1$, $T_1 = 1$, $g_1 = 0$. When $\chi_2 = 0$, $T_2 = -1$ we shall have

$$2\Omega^4 - \{r_1g_1'\,[s_1 + \frac{r_1^{2m}}{r_2^{2m}}(d_m + s_2)] - 2\,(r_2g_2' + 2g_2)\,(d_{-m} + s_2 - mg_2)\}\,\Omega^2 -$$

$$- r_1 g_1' (r_2 g_2' + 2g_2) \{s_1 (d_{-m} + s_2 - m g_2) + \frac{r_1^{2m}}{r_2^{2m}} [(m g_2 - s_2)(d_{-m} + s_2 + s_1) + (d_m + s_2)(d_{-m} + s_2 - m g_2)]\} = 0 \qquad (4.7)$$

where

$$d_n = 2n\,(n+1)\, r_2^{2n} \int_r^{r_2} g r^{-2n-1} dr, \qquad n = \pm m$$

In the absence of discontinuities in the density of current at the internal boundary $g_1' = 0$ and instead of (4.7) we obtain

$$\Omega^2 = -(r_2 g_2' + 2g_2)\{k r_0 h_2 + 2m\,(m-1)\, r_2^{-2m} \int_{r_1}^{r_2} g r^{2m-1} dr\} \qquad (4.8)$$

Hence it is clear how important is the assumption concerning continuity of the first derivatives of the functions $H_\varphi(r)$ and $H_z(r)$. In the presence of discontinuities in the distribution of current density there arise supplementary characteristic oscillations, which can be unstable.

The question of the absence of characteristic oscillations for a cylinder with g, h, g' and h' continuous everywhere can be considered in a more general form.

Since the terms in ξ_r drop out of the relations (1.9), (1.10), the dispersion equation is given by the vanishing of the determinant of the system (1.9), (1.10), in which the solutions of Equations (4.2), (4.3) have been substituted. From physical considerations it is obvious that the solution is $\mathbf{H}^* = 0$. Otherwise from condition (1.8) for an arbitrary frequency of oscillation we could determine the two remaining unknown constants occurring in the solution of the equation for Q, and there would exist oscillations for any specified frequency. In view of the fact that $\sigma(r_j) = 0$, from (4.1) we obtain the following supplementary conditions when $E(r_j) \neq 0$

$$\left(\frac{1}{r}\frac{\partial r H_\varphi^*}{\partial r} - \frac{im}{r} H_r^*\right)_{r=r_j} = 0, \qquad \left(\frac{\partial H_z^*}{\partial r} - ik H_r^*\right)_{r=r_j} = 0$$

The solution is $\mathbf{H}^* = 0$.

When $\mathbf{H}^* = 0$ from Equation (1.5) taken with (1.8) we obtain $Q = 0$, $\xi_r = 0$. In this case characteristic oscillations of a thin cylinder do not occur.

The equilibrium configuration with a discontinuity in the distribution of current density can be obtained by a passage to the limit from the continuous distribution. Then characteristic oscillations will be absent for any distribution of current density, which contradicts the conclusions derived above. It is necessary, however, to notice that for a sufficiently large value of the gradient of the field the magnetohydrodynamic equations become inapplicable, so that the proposed limiting transition is impermissible. In the presence of forces of surface tension a configuration with a discontinuity of conductivity at the boundary seems to be more realistic.

BIBLIOGRAPHY

1. Shafranov, V.D., Ob ustoichivosti tsilindricheskogo gazovogo provodnika v magnitnom pole (On the stability of a cylindrical gaseous conductor in a magnetic field). Atomnaia Energiia No. 5, p. 38, 1956.

2. Taylor, R.J., The influence of an axial magnetic field on the stability of a constricted gas discharge. *Proc. Phys. Soc.* Vol. 70, p. 1049, 1957.

3. Shafranov, V.D., Ob ustoichivosti plazmennogo shnura s raspredelennym tokom (On the stability of a plasma filament with a distributed current). *Akad. Nauk SSSR* Vol. 4, p. 61, 1958.

4. Vandakurov, Iu.V. and Lur'e, K.A., Ob ustoichivosti tsilindricheskogo plazmennogo provodnika s ob''emnymi tokami (On the stability of a cylindrical plasma conductor with space currents). *Zh. tek. fiz.* Vol. 29, p. 1170, 1959.

5. Vandakurov, Iu.V., K voprosu ob ustoichivosti plazmennogo tsilindra v sluchae neodnorodnogo raspredeleniia toka po secheniiu (On the question of stability of a plasma cylinder in the case of nonuniform distribution of current in its cross-section). *Dokl. Akad. Nauk SSSR* Vol. 143, p. 1078, 1962.

6. Suydam, B.R., *Proceedings of the Second United Nations Conference on the Peaceful Uses of Atomic Energy.* United Nations, Geneva, Vol. 31, p. 157, 1958.

7. Rosenbluth, M.N., Teoriia samoszhatogo razriada, ustoichivost' i nagrev (Theory of a self-compressed discharge, its stability and heating). Tr. II Mezhdunarodnoi konferentsii po mirnomu ispol'zovaniiu atomnoi energii (Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy). *Atomizdat* p. 55, 1959.

8. Taylor, R.J., Stability of twisted magnetic fields in a fluid of finite electrical conductivity. *Rev. Mod. Phys.* 32, 907, 1960.

9. Breus, S.N., Ob ustoichivosti zhidkogo tsilindra s tokom pri konechnoi provodimosti (On the stability of a thin cylinder carrying a current with finite conductivity). *Zh. tek. fiz.* Vol. 30, p. 1030, 1960.

10. Murty, G.S., Instability of a conducting fluid cylinder in the presence of an axial current, a longitudinal magnetic field and a coaxial conducting cylinder. *Arkiv för Fysik* 19, 483, 1961.

Translated by A.H.A.